The Isospin Splittings of Heavy-Light Quark System

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Abstract

The mass splittings of the pseudoscalar and vector D and B light-heavy quark systems have been calculated using the method of QCD sum rules. Electromagnetic, quark mass, and nonperturbative QCD effects are all included. The results are in good agreement with experiment. A measurement of isospin splitting for the vector B mesons would give valuable information about quark mass splittings.

The origin of mass differences in isospin multiplets has long been of great interest in nuclear and particle physics as a source of information about symmetry violations. Hadronic isospin violations are particularly important in that they arise from nonperturbative Quantum Chromodynamics (QCD) as well as quark mass differences (see Ref.[1] for a review of the early work in this area), and of course electromagnetic effects. Among the first applications of the method of QCD sum rules was the study of isospin violations in the $\rho - \omega$ system[2], in which it was recognized that the isospin splitting of the light-quark condensates can produce effects as large as the current-quark mass splittings and electromagnetic effects. Recently the QCD sum rule method has been used to study the neutron-proton mass difference[3, 4], the octet baryon mass splittings[5] and the mass differences in the charmed meson systems (the D and D* scalar and vector mesons)[6].

In the present work we study the mass splittings of the D,D*, B and B* systems using the QCD sum rule method. A main feature of the study is a consistent treatment of electromagnetic corrections within the framework of the method. Rather than make use of estimates of electromagnetic effects from quark models as used in previous work, or a phenomenological parametrization of electromagnetic effects as in Ref[4], we explicitly calculate the two-loop electromagnetic corrections as well as the perturbative and nonperturbative QCD processes including quark masses up to dimension eight. This is part of a study of light-heavy mesons[7] in which the accuracy of heavy quark effective theories is tested.

The isospin splittings are particularly interesting in that the electromagnetic effects and quark mass splittings enter with different relative signs in the charmed vs the bottom systems, magnifying the role of the isospin violation of the quark condensates. On the other hand, nonperturbative QCD mechanisms are expected to be less important for heavy quark systems, so that an accurate treatment of electromagnetic processes allows one to study light quark mass differences. That will be our main conclusion.

One can write the correlators in the QCD sum rule as

$$\Pi_t(p^2) = \Pi_0(p^2) + m_q \Pi_m(p^2) + \frac{\alpha_e}{4\pi}\Pi_{em}(p^2). \tag{1}$$

For $\Pi_0(p^2)$, the leading term for the light heavy quark system, the isospin violation comes from the isospin splitting of the quark condensates in the non-perturbative power correction, which has been given in previous publications[7, 8]. The second term in Eq. 1 is the light quark mass, m_q, expansion of the total correlator; and the third term corresponds to the electromagnetic corrections, which have been not treated consistently in the framework of the QCD sum rule method in the literature. The m_q expansion is extended to dimension 5 in our calculation, and furthermore, we also include the one-loop corrections to the quark condensate, which we believe is significant. Thus, $\Pi_m(p^2)$ in Eq. 1 can be written as

$$\Pi_m(q^2) = C_I I + C_3\langle\bar{q}q\rangle + C_5\langle\bar{q}(\sigma\cdot G)q\rangle \tag{2}$$

where the coefficient C_I represents the perturbative contributions, whose Feynman diagrams are shown in Fig. 1. Based on the results in Ref. [9], we find the m_q expansion to order α_s is

$$ImC_I^{p.s} = \frac{3M_Q}{4\pi}(1-x)\left[1+\frac{4\alpha_s}{3\pi}\left(f(x)+\frac{3}{4}x+\frac{3}{2}x\ln\left(\frac{x}{1-x}\right)\right)\right] \tag{3}$$

for the pseudoscalar and

$$ImC_m^v = ImC_m^{p.s} - \frac{\alpha_s M_Q}{\pi^2}(1-x) \tag{4}$$

for the vector correlator, where $x = \frac{M_Q^2}{p^2}$,

$$f(x) = \frac{9}{4} + 2l(x) + \ln(x)\ln(1-x) + \left(\frac{5}{2} - x - \frac{1}{1-x}\right)\ln(x) - \left(\frac{5}{2} - x\right)\ln(1-x) \tag{5}$$

and $l(x) = -\int_0^x \ln(1-y)\frac{dy}{y}$ is the Spencer function. Particular attention has been paid to the m_q expansion of the quark condensate since we find that it plays an important in the heavy-light quark systems. In particular, we have considered the one-loop corrections to the quark condensate whose Feynman diagrams are shown in Fig. 2-a, b, c, and d, the result to the order α_s is

$$\begin{aligned} C_3^{p.s} = & -\frac{1}{2}\frac{1}{p^2(1-x)}\left\{\frac{x}{1-x} - 1\right. \\ & + \left.\frac{4\alpha_s}{3\pi}\left[\frac{3x}{4(1-x)}\left(7+2\ln\left(\frac{x}{x-1}\right)\right) - x\left(2-(3+2x)\ln\left(\frac{x}{x-1}\right)\right)\right]\right\} \end{aligned} \tag{6}$$

for the pseudoscalar and

$$\begin{aligned} C_3^{v} = & -\frac{1}{2}\frac{1}{p^2(1-x)}\left\{\frac{x}{1-x}\right. \\ & + \left.\frac{4\alpha_s}{3\pi}\left[\frac{x}{4(1-x)}\left(1-6\ln\left(\frac{x}{x-1}\right)\right) + 2x\left(1-x\ln\left(\frac{x}{x-1}\right)\right) + \frac{1}{2}\right]\right\} \end{aligned} \tag{7}$$

for the vector currents. The details of the one loop calculations will be given later[7]. However, we find that the m_q expansions of the one loop corrections to the quark condensate are finite after the mass renormalization. The m_q expansion for the quark gluon condensate corresponding to Fig. 2-e and f is

$$C_5^{p.s} = \frac{x}{p^4(1-x)^3}\left(\frac{3x}{2(1-x)} - 1\right) \tag{8}$$

and

$$C_5^{v} = \frac{1}{p^4(1-x)^2}\left(\frac{3x^2}{2(1-x)^2} + \frac{5x}{6(1-x)} - \frac{5}{12}\right) \tag{9}$$

for the pseudoscalar and vector current, respectively.

For the electromagnetic corrections, we write $\Pi_{e.m}(q^2)$ as

$$\Pi_{e.m}(q^2) = D_I I + D_3\langle\bar{q}q\rangle + \dots \tag{10}$$

The coefficient D_I in Eq. 10 is a two-loop perturbative contribution which replace the gluon lines in Fig. 1 by the corresponding photon lines. Therefore, the imaginary part of the coefficient D_I can be obtained from the two-loop integral[9] in QCD for the charge neutral current, where the charges for the heavy quark e_Q and the light quark e_q are equal with opposite sign:

$$ImD_I^{p.s} = \frac{3e_q^2 M_Q^2}{2\pi}(1-x)^2 f(x) \tag{11}$$

and

$$ImD_I^{v} = \frac{e_q^2 p^2}{2\pi}(1-x)^2\Big[(2+x)(1+f(x)) - (3+x)(1-x)\ln\left(\frac{x}{1-x}\right) - \frac{2x}{(1-x)^2}\ln(x) - 5 - 2x - \frac{2x}{1-x}\Big] \tag{12}$$

for the pseudoscalar and the vector currents, where $f(x)$ is given by Eq. 5. The calculation of D_I for the charge currents is more complicated. Since the charges for the heavy and light quarks are not the same, the Ward identity can not be used here; thus, one should do the wavefunction and mass renormalizations at the same time. We can write the imaginary part of the correlator for the charged currents as

$$ImD_I^{+} = -e_Q e_q D^{+} + (e_Q^2 + e_Q e_q)D_Q + (e_q^2 + e_Q e_q)D_q \tag{13}$$

where D^+ can be obtained from Eqs. 10 and 11;

$$D^{+} = D_I^{p.s,v}/e_q^2. \tag{14}$$

$D_{Q,q}$ is the integral for the self energy diagrams for the heavy and the light quarks. The corresponding imaginary parts are

$$ImD_Q^{p.s} = \frac{3p^2}{2\pi}\Big[\frac{1}{3}(1-x)^4\left(1+\frac{1}{5}(1-x)\right) - \frac{17x}{4}\left(\ln(x)+1-x\right) -x(1-x)^2\left(\frac{15}{8} - \frac{1}{4}\ln\left(\frac{M_Q^2}{\mu^2}\right) + \frac{5}{4(1-x)}\ln(x) - \ln\left(\frac{x}{1-x}\right)\right)\Big], \tag{15}$$

$$ImD_Q^{v} = \frac{3p^2}{2\pi}\Big[\frac{1}{9}(1-x)^4\left(1+\frac{1}{5}(1-x)\right) - \frac{5x}{12}\left((1-x)^2 + 2(\ln(x)+1-x)\right) -\frac{1}{3}(1-x)^2(2+x)\left(\frac{5}{8} - \frac{1}{4}\ln\left(\frac{M_Q^2}{\mu^2}\right) + \frac{5}{4(1-x)}\ln(x) - \ln\left(\frac{x}{1-x}\right)\right) -\frac{7x}{12}(2+x)(1-x+\ln(x))\Big], \tag{16}$$

$$ImD_q^{p.s} = \frac{3p^2}{2\pi}\Big[\frac{1}{6}(1-x)^4\left(1+\frac{1}{5}(1-x)\right)+\frac{5x}{4}\left(\frac{1}{2}(1-x)^2+x(1-x+\ln(x))\right)$$
$$-\frac{x}{4}(1-x)^2\left(2-\ln\left(\frac{M_Q^2}{\mu^2}\right)+2\ln\left(\frac{x}{1-x}\right)-\frac{1}{1-x}\ln(x)\right)\Big] \quad (17)$$

and

$$ImD_q^{v} = \frac{3p^2}{2\pi}\Big[\frac{(1-x)^4}{18}\left(1+\frac{(1-x)}{5}\right)+\frac{(5x+2)}{12}\left(\frac{(1-x)^2}{2}+x(1-x+\ln(x))\right)$$
$$-\frac{(2+x)}{12}(1-x)^2\left(2-\ln\left(\frac{M_Q^2}{\mu^2}\right)+2\ln\left(\frac{x}{1-x}\right)-\frac{1}{1-x}\ln(x)\right)\Big] \quad (18)$$

for the pseudoscalar and vector currents, respectively.

The D_3 in Eq. 10 represents the one-photon-loop corrections to the quark condensate, whose Feynman diagrams are shown in Fig. 2. For the charge-neutral systems, with the appropriate change in coupling constants, the results are the same as the one-gluon-loop corrections to the quark condensate which have been calculated explicitly[7]. We find

$$D_3^{p.s} = e_q^2\frac{x}{M_Q(1-x)}\left[2+3\ln\left(\frac{M_Q^2}{\mu^2}\right)-6(1-x)\left(1+x\ln\left(\frac{x-1}{x}\right)\right)\right] \quad (19)$$

and

$$D_3^{v} = e_q^2\frac{x}{M_Q(1-x)}\left[3\ln\left(\frac{M_Q^2}{\mu^2}\right)-2+2(1-x)\left(1+x\ln\left(\frac{x-1}{x}\right)\right)\right] \quad (20)$$

for the pseudoscalar and vector respectively. We set $\mu = M_Q$ in our numerical evaluations. There are different expressions for the charged D_3's. We find

$$D_3^{p.s} = \frac{2}{9}\frac{x}{M_Q(1-x)}\left[1+6\ln\left(\frac{M_Q^2}{\mu^2}\right)+6(1-x)\left(1+x\ln\left(\frac{x-1}{x}\right)\right)\right] \quad (21)$$

and

$$D_3^{v} = \frac{2}{9}\frac{x}{M_Q(1-x)}\left[5+6\ln\left(\frac{M_Q^2}{\mu^2}\right)-2(1-x)\left(1+x\ln\left(\frac{x-1}{x}\right)\right)\right] \quad (22)$$

for the charged pseudoscalar and vector currents of D mesons, and

$$D_3^{p.s} = \frac{2}{9}\frac{x}{M_Q(1-x)}\Big[4+21\ln\left(\frac{M_Q^2}{\mu^2}\right)+9(1+x)\ln\left(\frac{x-1}{x}\right)$$
$$+6(1-x)\left(1+x\ln\left(\frac{x-1}{x}\right)\right)\Big] \quad (23)$$

and

$$D_3^{v} = \frac{2}{9}\frac{x}{M_Q(1-x)}\Big[8+21\ln\left(\frac{M_Q^2}{\mu^2}\right)+9(1+x)\ln\left(\frac{x-1}{x}\right)$$
$$-2(1-x)\left(1+x\ln\left(\frac{x-1}{x}\right)\right)\Big] \quad (24)$$

for the charged pseudoscalar and vector currents of the B mesons. There are also corresponding one-photon-loop corrections to the quark-gluon condensates and higher dimension power corrections. Whether they are important numerically are remained to be studied. As we indicate later, the one photon loop corrections to the quark-gluon condensate might be important.

Defining the quantity $\omega^2 = M_Q^2 - p^2$ for the Borel transformation, where ω^2 measures the off-shell effects of the heavy quarks, we find

$$f_p^2 \frac{M_p^4}{M_Q^3} e^{-\frac{M_p^2 - M_Q^2}{\omega_B^2}} = \Pi_t^p(\omega_B^2) \tag{25}$$

and

$$f_v^2 \frac{M_v^2}{M_Q} e^{-\frac{M_v^2 - M_Q^2}{\omega_B^2}} = \Pi_t^v(\omega_B^2) \tag{26}$$

for the pseudoscalar and vector current. The correlator $\Pi_t(\omega_B^2)$ can be divided into the perturbative and nonperturbative parts:

$$\Pi_t(\omega_B^2) = \int_0^{\omega_0^2} Im C_I^t(\omega^2) e^{-\frac{\omega^2}{\omega_B^2}} d\omega^2 + \Pi_{np}(\omega_B^2), \tag{27}$$

where the perturbative term is written in dispersion integral form, and the nonperturbative correlator $\Pi_{np}(\omega_B^2)$ comes from the standard Borel transformations of the terms associated with the condensates. The masses M_p and M_v in Eqs. 25 and 26 can be obtained from

$$M^2 = M_Q^2 + \omega_B^4 \frac{d \ln\left(\Pi_t(\omega_B^2)\right)}{d\omega_B^2}. \tag{28}$$

The mass differences between different isospin states are obtained by adjusting the parameters ω_0^2 in Eq. 27 to insure that the mass difference is independent of Borel parameter ω_B^2. The range of ω_B^2 is chosen in such way so that the the continuum and the nonperturbative contributions are minimum.

There are two parameters that are important for the isospin mass splittings: the up and down quark mass difference, $m_d - m_u$, and the isospin splitting of the quark condensate, which is defined by the parameter γ;

$$\gamma = \frac{\langle \bar{d}d \rangle}{\langle \bar{u}u \rangle} - 1. \tag{29}$$

The quark mass difference has been estimated via broken chiral symmetry[1], while the parameter γ in various estimates ranges from -0.0079[4] to -0.002[5, 6]. Physically, the sensitivity of the isospin mass splittings to the parameter γ measures the strength of the nonperturbative effects on the light-heavy quark system. To examine the sensitivity of the mass splittings on the parameters γ, we calculated the mass splittings for different γ's in our framework, with the quark mass difference $m_d - m_u$ fixed at

3.8 MeV. Our result for different γ and $m_d - m_u$ fits approximately to the empirical formula [all mass differences are given in MeV]

$$M_{D^\pm} - M_{D^0} = 0.1 - 50\gamma + 1.1(m_d - m_u), \tag{30}$$

$$M_{D^{*\pm}} - M_{D^{*0}} = -0.9 - 10\gamma + 1.0(m_d - m_u), \tag{31}$$

for D and D^* mesons and

$$M_{B^\pm} - M_{B^0} = 1.45 - 1.16(m_d - m_u) \tag{32}$$

$$M_{B^{*\pm}} - M_{B^{*0}} = 1.36 - 1.28(m_d - m_u) \tag{33}$$

for B and B^* mesons. Comparing to the corresponding formula for the neutron-proton mass difference[4], neglecting electromagnetic corrections,

$$M_n - M_p = -416\gamma + 0.197(m_d - m_u), \tag{34}$$

we find that the isospin mass splittings are much less sensitive to the γ for the D mesons, and the mass splittings for the B systems is almost independent of γ. This result differs from the conclusion[6] of Eletsky and Ioffe who found a stronger dependence of isospin mass splittings for the D mesons on the parameter γ. The difference is due to their neglect of higher dimension terms, especially the quark gluon condensate in $\Pi_0(p^2)$ and $\Pi_m(p^2)$ of Eq. 1, which is very important in our calculations. The weak dependence of the isospin splittings on the isospin splitting of the quark condensate is consistent with conclusions of heavy quark symmetry that the nonperturbative effects become less important for heavier quark systems. Therefore, the heavy-light quark systems provide an excellent testing ground for the up and down quark mass difference if electromagnetic corrections are calculated accurately.

For $\gamma = -0.0079$ and $m_d - m_u = 3.8$ MeV, we have

$$M^\pm - M^0 = \begin{cases} 4.68 & \text{for D} \\ 2.94 & \text{for } D^* \end{cases} \tag{35}$$

and

$$M^\pm - M^0 = \begin{cases} -3.0 & \text{for B} \\ -3.5 & \text{for } B^* \end{cases} . \tag{36}$$

This is in excellent agreement with data ($M_{D^\pm} - M_{D^0} = 4.7 \pm 0.7$ MeV and $M_{D^{*\pm}} - M_{D^{*0}} = 2.9 \pm 1.3$ MeV[10]) for D mesons, while the magnitude of our result for B mesons is quite large comparing to the data -0.1 ± 0.8 MeV[10]. One of the important features of our calculation is the differential vector-pseudoscalar isospin mass splittings $\Delta(M)$;

$$\Delta(M) = (M^{*\pm} - M^{*0}) - (M^\pm - M^0), \tag{37}$$

which provides an very important probe of the hyperfine splittings of the heavy-light quark potential. Our result indicates that $\Delta(M)$ is less dependent on the quark mass difference $m_d - m_u$, and from Eqs. 35 and 36, we have

$$\Delta(M) = \begin{cases} -1.7 & \text{for D mesons} \\ -0.5 & \text{for B mesons} \end{cases} \tag{38}$$

for $m_d - m_u = 3.8$ MeV. This is in good agreement with recent studies by Cutkosky and Geiger[11]. In particular, the $\Delta(M)$ for B mesons is negative for positive $m_d - m_u$, which is opposite to the quark model predictions and several other investigations[12]. A measurement of the isospin mass splittings for the vector B mesons will provide a crucial test in this regard. A study in chiral perturbation theory finds[13] that the contributions from quark mass difference only gives 0.3 MeV for D mesons, and the rest might comes from the electromagnetic corrections. Our calculation is consistent with this conclusion, and furthermore we also find that there is only a small contribution from the difference of the quark condensates to $\Delta(M)$.

We find the electromagnetic corrections to the isospin splittings to D and D^* mesons are quite small. They contribute less than 1 MeV. As a result, our splittings in the charmed systems are too small in comparison with experiment for parameters giving good fits to the B splittings. One source of this discrepancy might be that the one photon loop corrections to the quark gluon condensate are needed, since we find that the quark gluon condensate is as important as the quark condensate in evaluating the masses of the heavy ligth systems. This might both help to explain the electromagnetic corrections are small for the D mesons and improve our fit to the B mass splitting with $m_d - m_u$ about 3.0 MeV. This is a somewhat subtle calculation, which we do not attempt here.

In summary, we show how to include the electromagnetic corrections in the QCD sum rule analysis for the light-heavy quark systems. This approach can also be extended to the study of the neutron-proton mass difference as well as other isospin mass splittings. Our results show that the mass splittings are not sensitive to the isospin splitting of the quark condensate, and thus heavy-light quark systems are an ideal place to study the up and down quark mass differences. We expect the method will give a very good qualitative even quantitative descriptions of the isospin-splittings for the light-heavy quark systems. Our result for $\Delta(M)$ is in good agreement with recent phenomenological studies by Cutkosky and Geiger. Further investigation of the electromagnetic effects of higher dimensions terms is needed, and this is in progress. Our investigation shows that the isospin mass splittings of the light heavy quark systems provide an very important opportunity to study the light quark masses and the hyperfine interactions of the quark-quark potential.

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Figure Captions

1. Processes for two loops correction.
2. Contribution from nonperturbative processes.